Physics 4251 / Fall 2017
Problem Set 3 Solutions
45 points in total

Problem 1: Force of an entropic spring

(a, 5) What is the expected length when there is no applied force and when $T = 0$?

For the case of no applied force, the spring is just a freely jointed chain. Thus, $\langle L \rangle = 0$. When $T = 0$, the entropic term contributes no resistance and, thus, the force extends the chain completely, making $\langle L \rangle = Na$.

(b, 5) Derive an expression for S.

$$S = k \ln(W) = k \ln(\frac{N!}{n_r!(N-n_r)!}) \tag{1}$$
$$\approx k[(N \ln N - N) - (n_r \ln n_r - n_r) - ((N-n_r)\ln(N-n_r) - (N-n_r))] \tag{2}$$
$$= k[N \ln N - (N-n_r)\ln(N-n_r) - n_r \ln n_r] \tag{3}$$

(c, 10) Find the length that minimizes G.

$$\frac{\partial G}{\partial L} = -F - T(\frac{\partial S}{\partial n_r}\frac{\partial n_r}{\partial L}) = 0 \tag{4}$$

$$L = (n_r - n_l)a = (n_r - (N - n_r))a = (2n_r - N)a \rightarrow n_r = \frac{1}{2}(\frac{L}{a} + N) \tag{5}$$

$$\frac{\partial n_r}{\partial L} = \frac{1}{2a} \tag{6}$$

$$\frac{\partial S}{\partial n_r} = k[0 + (\ln(N-n_r) + 1) - (1 + \ln n_r)] = k \ln(\frac{N-n_r}{n_r}) \tag{7}$$

$$F = -\frac{kT}{2a} \ln(\frac{N-n_r}{n_r}) \rightarrow n_r = \frac{N}{1 + e^{-2aF/kT}} = \frac{1}{2}(\frac{L}{a} + N) \tag{8}$$

$$L = aN\frac{1 - e^{-2aF/kT}}{1 + e^{-2aF/kT}} = aN \tanh(\frac{aF}{kT}) \tag{9}$$

Problem 2: The cleavage mechanism of the autocatalytic transporter EspP

(a, 3). What amino acid was used in place of Asn1023?

Serine was used in this structure.

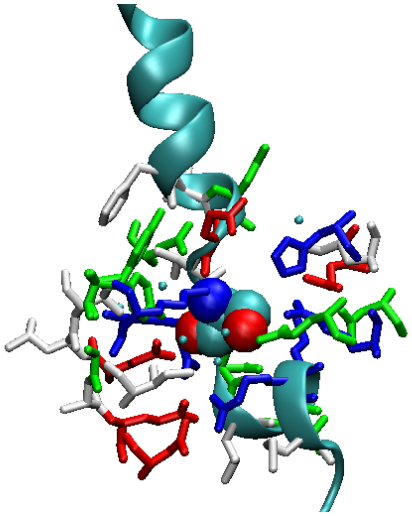

Figure 1:

(b, 6). Make a figure of the active site and describe anything that may be unusual about it.

See Fig. 1. For example, there are a large number of charged residues, particularly acidic (negatively charged), near the active site.

(c, 4). How many negatively/positively charged residues are in EspP?

Using the atom selection ‘‘`resname LYS ARG and name CA`’’, we see there are 34 positive residues (note that histidine is typically neutral, but if you included it you would get 45). Similarly, with ‘‘`resname ASP GLU and name CA`’’, we find 39 negative residues.

(d, 6). Pick three residues from the pKa calculation and give their probability of being protonated.

$$\mathrm{pH} = \mathrm{pK}_a + \log(\frac{[\mathrm{A}^-]}{[\mathrm{HA}]}) \rightarrow p_{\mathrm{prot.}} = \frac{[\mathrm{HA}]}{[\mathrm{HA}] + [\mathrm{A}^-]} = \frac{1}{1 + [\mathrm{A}^-]/[\mathrm{HA}]} = \frac{1}{1 + 10^{(\mathrm{pH} - \mathrm{pK}_a)}} \tag{10}$$

1. Negative: Asp1014, $\mathrm{pK}_a = 8.08$, $p_{\mathrm{prot.}} = 0.92$ (note that this is VERY unusual).
2. Positive: Lys1001, $\mathrm{pK}_a = 10.29$, $p_{\mathrm{prot.}} = 0.999$.
3. Histidine: His1062, $\mathrm{pK}_a = 2.63$, $p_{\mathrm{prot.}} = 4.3 \times 10^{-5}$.

(e, 6). What residues are predicted to be in a non-standard protonation state? Where are they located? Which is closest? Does it make sense to alter its state and why or why not?

An easy way to identify those residues that may be in non-standard states is to look at those basic (positive) residues with a pK_a below 7.0 and those acidic (negative) residues above 7.0 ($p_{\mathrm{prot.}} = 0.5$). Using this metric, the acidic residues Asp1014 (8.08), Glu1172 (8.23), and Glu1279 (7.38) are all

predicted to be protonated. The latter two are near the active site, with Glu1172 being the closest. Given the close distance between the carbonyl O of Ser1023 and the O of Glu1172 (2.765 Å), it makes sense that it should be protonated to prevent repulsion of these two negative atoms. See Fig. 2.

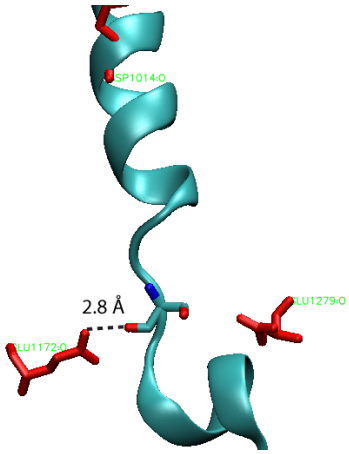

Figure 2: