

**Physics 4251 / Fall 2017**  
**Problem Set 5 Solutions**  
**55 points in total**

**Problem 1 (30 points): Drug permeability**

a. (5) Determine how  $J$  depends on  $\mu$ . How does this relate to diffusion in the presence of a force as discussed in class?

$$J = -D \frac{dc}{dz} = -D \frac{d}{dz} e^{(\mu-\mu_0)/kT} = -\frac{D}{kT} e^{(\mu-\mu_0)/kT} \frac{d\mu}{dz} \quad (1)$$

where it was assumed that  $d\mu_0/dz = 0$ . In the presence of a force, the current density related to that force is  $J = (F/\gamma)c$ . The expression just derived for  $J$  can also be written as

$$J = -\frac{D}{kT} \left( \frac{d\mu}{dz} \right) c = -\frac{c}{\gamma} \frac{d\mu}{dz}, \quad (2)$$

which is identical if  $F = -\frac{d\mu}{dz}$ . Thus, we see that all chemical gradients are, in fact, due to a force, including one associated with the chemical potential.

b. (5) Simplify Fick's first law in terms of the potential  $G(z)$ .

$$J = -D \frac{dc}{dz} + \frac{F(z)}{\gamma} c = -D(z) \frac{dc}{dz} + \frac{D(z)}{kT} c \left( -\frac{dG(z)}{dz} \right) = -D(z) \left( \frac{dc}{dz} + \frac{c}{kT} \frac{dG(z)}{dz} \right) \quad (3)$$

$$\rightarrow \frac{J e^{G(z)/kT}}{D(z)} = - \left( \frac{dc}{dz} e^{G(z)/kT} + \frac{c}{kT} \frac{dG(z)}{dz} e^{G(z)/kT} \right) = -\frac{d}{dz} \left( c(z) e^{G(z)/kT} \right) \quad (4)$$

c. (5) Integrate the equation from part (b) under steady-state conditions.

$$J \int_{z_1}^{z_2} \frac{e^{G(z)/kT}}{D(z)} dz = - \int_{z_1}^{z_2} \frac{d}{dz} \left( c(z) e^{G(z)/kT} \right) dz \quad (5)$$

$$J \int_{z_1}^{z_2} \frac{e^{G(z)/kT}}{D(z)} dz = c(z_1) e^{G(z_1)/kT} - c(z_2) e^{G(z_2)/kT} \quad (6)$$

$$J \int_{z_1}^{z_2} \frac{e^{G(z)/kT}}{D(z)} dz = -e^{G(z_1)/kT} \Delta c \quad (7)$$

d. (5) Express the permeability  $P$  in terms of the free energy  $G$ .

$$P = -J/\Delta c = \left( \int_{z_1}^{z_2} \frac{e^{G(z)/kT}}{D(z)} dz \right)^{-1} e^{G(z_1)/kT} = \left( e^{-G(z_1)/kT} \int_{z_1}^{z_2} \frac{e^{G(z)/kT}}{D(z)} dz \right)^{-1} \quad (8)$$

$$= \left( \int_{z_1}^{z_2} \frac{e^{\Delta G(z)/kT}}{D(z)} dz \right)^{-1}. \quad (9)$$

e-f. (10) Calculate  $P$  in the traditional units of cm/s using the provided data on urea. The experimental value is  $\log(P) = -5.4$ ; how does the one you calculated compare?

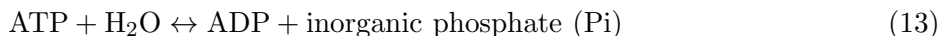
Applying the formula to the provided data and integrating gives  $P = 1/(6.7663 \times 10^{10}) = 1.478 \times 10^{-11}$  nm/ps (see HW5-P1-solution.ipynb on the class Github). To convert the units, note that  $1 \text{ nm/ps} = 10^5 \text{ cm/s}$ ; thus,  $P = 1.478 \times 10^{-6} \text{ cm/s}$  and  $\log(P) = -5.83$ . This is only 8% off from the experimental value.

**Problem 2 (20 points): Energy from ATP using  $\Delta G$  and the standard state**

$$K_{\text{eq}} = \frac{[C][D]}{[A][B]}, \quad (10)$$

$$\Delta G = \Delta G^\circ + kT \ln K_{\text{eq}}. \quad (11)$$

$$\Delta G = \Delta G^\circ + RT \ln\left(\frac{[C][D]}{[A][B]}\right) \quad (12)$$



(a, 5) After an equilibrium experiment at body temperature, we find that  $K_{\text{eq}} = 130,000$ . What is  $\Delta G^\circ$ ?

$$\Delta G^\circ = -kT \ln K_{\text{eq}} = -1.99 \times 10^{-3} \frac{\text{kcal}}{\text{mol} \cdot \text{K}} \times 310 \text{ K} \times \ln(1.3 \times 10^5) = -7.26 \text{ kcal/mol} = -30.4 \text{ kJ/mol} \quad (14)$$

For a single ATP,  $\Delta G^\circ = -30.4 \text{ kJ/mol}/N_A = 5.04 \times 10^{-23} \text{ kJ}$ .

(b, 5) Let's say that  $[\text{ATP}] = 10 \text{ mM}$ ,  $[\text{ADP}] = 100 \mu\text{M}$ , and  $[\text{Pi}] = 1 \text{ mM}$ . What is the free energy, or the energy available from the hydrolysis of 1 mole of ATP? For an individual ATP?

$$\Delta G = (-7.26 \text{ kcal/mol}) + 1.99 \times 10^{-3} \frac{\text{kcal}}{\text{mol} \cdot \text{K}} \times 310 \text{ K} \times \ln\left(\frac{100 \mu\text{M} \cdot 1 \text{ mM}}{10 \text{ mM} \cdot 1 \text{ M}}\right) = \quad (15)$$

$$-14.4 \text{ kcal/mol} = -60.3 \text{ kJ/mol}. \quad (16)$$

For a single ATP molecule,  $\Delta G = (-14.4 \text{ kcal/mol})/N_A = -2.4 \times 10^{-23} \text{ kcal} = -10.0 \times 10^{-23} \text{ kJ}$ .

(c, 5) Now let  $[\text{ATP}] = 2 \text{ mM}$ ,  $[\text{ADP}] = 1 \text{ mM}$ , and  $[\text{Pi}] = 2 \text{ mM}$ . What is the free energy, or the energy available from the hydrolysis of an individual ATP?

$$\Delta G = (-7.26 \text{ kcal/mol}) + 1.99 \times 10^{-3} \frac{\text{kcal}}{\text{mol} \cdot \text{K}} \times 310 \text{ K} \times \ln\left(\frac{1 \text{ mM} \cdot 2 \text{ mM}}{2 \text{ mM} \cdot 1 \text{ M}}\right) = \quad (17)$$

$$-11.5 \text{ kcal/mol} = -48.1 \text{ kJ/mol}. \quad (18)$$

For a single ATP molecule,  $\Delta G = (-11.5 \text{ kcal/mol})/N_A = -1.9 \times 10^{-23} \text{ kcal} = -7.9 \times 10^{-23} \text{ kJ}$ .

*(d, 5) Express 100 pN-nm (roughly the amount of energy released with splitting one ATP) in units that allow you to compare this value to the free energy. Are they consistent?*

*As you can see, the amount of energy released by ATP depends on the cells [ADP] and [Pi], which vary from cell to cell; hence, ATP is 80-100 pN-nm.*

Working in units of kcal,  $100 \text{ pN-nm} = 10^{-22} \text{ kJ} = 2.4 \times 10^{-23} \text{ kcal}$ . The numbers are similar, with that from (b) being about 100 pN-nm and that from (c) being about 80 pN-nm.