Waves and Optics

A wave is a disturbance that propagates in time; it satisfies the equation

$$\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2} \quad (1D)$$

Solutions include any function $f(x \pm vt)$ traveling waves. This leads to the principle of superposition: for any two solutions $f(x,t)$ and $g(x,t)$, $f+g$ is also a solution.

Standing waves look like $f(x,t) = A(x)\,B(t)$

For example, $f(x,t) = \cos x \cos vt = \frac{1}{2}\left(\cos(x+vt) + \cos(x-vt)\right)$

GRE 1) Sound waves in air can be described by the equation $\frac{\partial^2 p}{\partial x^2} = K^2 \frac{\partial^2 p}{\partial t^2}$ where p is the pressure deviation and K is a constant. The speed of sound is

~~A) $1/K^2$~~ $\boxed{\text{B) } 1/K}$ C) $\sqrt{K}$ D) K E) K^2

GRE 3) Let $f(x,t)$ and $g(x,t)$ be two traveling ~~waves~~ solutions to the homogeneous wave equation. Which of the following are true?

I. $f+g$ solves the wave equation

II. fg solves the wave equation

III. $2f-3g$ solves it.

A) I B) II C) III $\boxed{\text{D) I and III}}$ E) II and III

Solutions to the wave equation can all be written as linear comb.s of

$$f(x,t) = A\cos(kx - \omega t + \delta) \quad \text{(thanks to Fourier)}$$

A is amplitude, k: wave number, ω: angular frequency, δ: phase

$\lambda = 2\pi/k$; $T = 2\pi/\omega$; $\omega = 2\pi f$

$\omega = vk \rightarrow$ dispersion relation (linear case) phase velocity : ω/k (same in linear case, like light in vacuum

group velocity : $d\omega/dk$ ← speed of information (phase > c is possible)

Because v_{phase} is a function of k, it is wavelength dependent (light is dispersed in prisms, for example.

wave packet

GRE #16 from test: Two ident. sinusoidal waves travel in opposite directions in a 15-m wire, producing a standing wave. The traveling waves have a speed of 12 m/s and the standing wave has six nodes, including those at the two ends. What is the wavelength and frequency?

$\lambda = \underline{\underline{2}} \cdot 3 = \underline{6\ m}$ $\quad v = \omega/k = \frac{2\pi f}{2\pi/\lambda} = \lambda f = 12\,\frac{m}{s} \rightarrow \underline{f = 2\ Hz}$

Interference

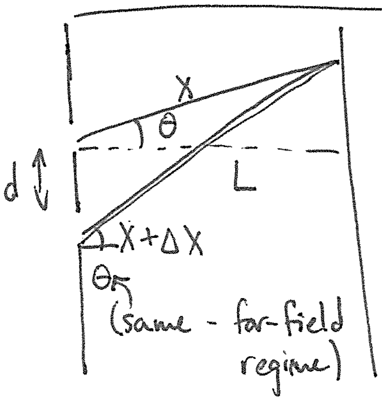
Consider $f(x,t) = A\cos(kx - \omega t)$ $\quad f + g = 0!$

$g(x,t) = A\cos(kx - \omega t + \pi)$ (destructive)

constructive : phase difference of $2m\pi$

destructive : " " " $(2m+1)\pi$

double-slit interference



single, monochromatic source, two slits act as two point sources

$\Delta x \approx d \sin\theta$

$f(x+\Delta x, t) = A\cos(k(x+\Delta x) - \omega t) = A\cos(kx - \omega t + k\Delta x)$

$\delta = k\Delta x$ (phase shift)

maxima: $d \sin\theta = m\lambda$ (constructive) remember! $\theta = 0 \to$ max
minima: $d \sin\theta = (m + \frac{1}{2})\lambda$ (destructive) so m, not $m + \frac{1}{2}$

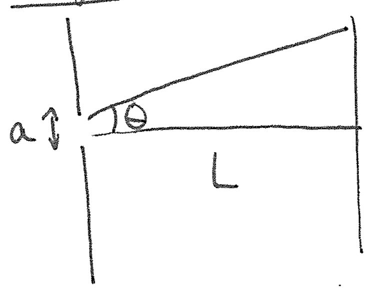
GRE #8) Monochromatic light (λ) is directed at two slits w/spacing d. If the same light is directed at a different two-slit arrangement with separation d', the position of the 3rd interference minimum corresponds to the position of the old 2nd interference max. after the central maximum. What is d' in terms of d and λ?

A) $4d/5$ B) $4\lambda^2/5d$ C) d $\boxed{\text{D) } 5d/4}$ E) $5d^2/4\lambda$

3rd min $\to$ $m = \underline{2}$ $d' \sin\theta = \frac{5}{2}\lambda \to \sin\theta = \frac{5\cancel{\lambda}}{2d'} = \frac{2\cancel{\lambda}}{d} \to d' = \frac{5}{4}d$

2nd max after central $\to m = 2$ $d \sin\theta = 2\lambda \to \sin\theta = \frac{2\lambda}{d}$

Single-slit diffraction



minima: $a \sin\theta = m\lambda$

leads to Rayleigh criterion for resolution of two distant point sources: $a \sin\theta = 1.22\lambda$ or $\Delta l = \frac{1.22\lambda L}{a}$

if $\theta < \theta_{crit}$, maxima of two sources overlap, blurring the image

GRE #14 (test): Observer looks through slit of width 5×10^{-4} m at two lanterns 1 km away. The lanterns emit light of wavelength 5×10^{-7} m. The min. separation at which they can be resolved is most nearly

A) 0.01m B) 0.1m $\boxed{\text{C) 1 m}}$ D) 10 m E) 100 m

$$\Delta l = \frac{1.22\lambda L}{a} = 1.22 \cdot \frac{\cancel{5}\times\cancel{10^{-7}}\,m \cdot \cancel{10^{3}}\,m}{\cancel{5}\times\cancel{10^{-4}}\,m} = 1.22\,m$$

Geometric optics

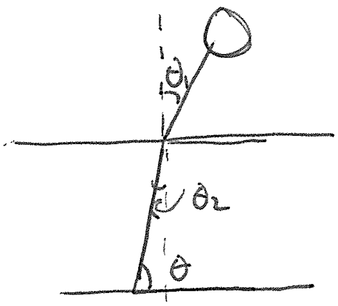
reflection : $\theta_i = \theta_r$ (incidence angle equals angle of reflection)

refraction : $n_1 \sin\theta_1 = n_2 \sin\theta_2$ (Snell's Law)

apply to any and all wave phenomena, e.g., sound, water, etc.

* angle is w/ respect to the normal

GRE #11) A person at the bottom of a pool looking up at the sky observes the sun at an angle θ from the horizon. What is the true angle of the sun, in terms of the index of refraction, n, of the water?



$\cancel{n_1}^{1} \sin\theta_1 = n_2 \sin\theta_2$

$\sin(90-\theta') = n_2 \sin(90-\theta)$

$\hookrightarrow \cos\theta' = n_2 \cos\theta \rightarrow \theta' = \cos^{-1}(n_2\cos\theta)$

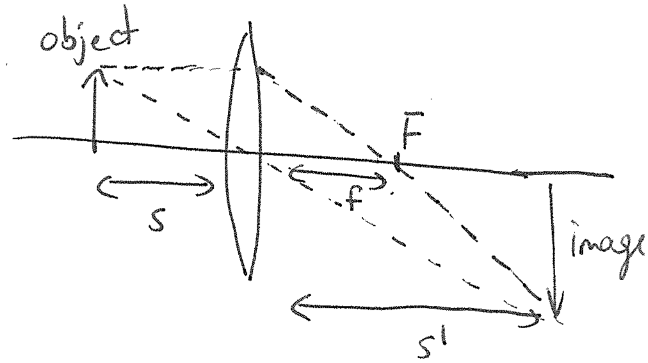
$\boxed{A)\ \cos^{-1}(n\cos\theta)}$ B) $\sin^{-1}(n\cos\theta)$ C) $\sin^{-1}(n\sin\theta)$

D) $\cos^{-1}(\cos\theta/n)$ E) $\sin^{-1}(\sin\theta/n)$

Lenses and Mirrors

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

f = focal length
s = object
s' = image



for mirrors, $f = R/2$ (radius of curvature)

convex lenses

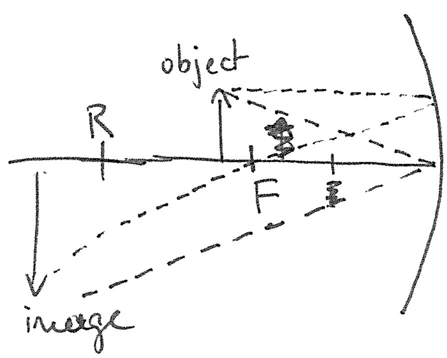
concave mirrors

$\rightarrow f > 0$

concave lenses

convex mirrors

$\rightarrow f < 0$



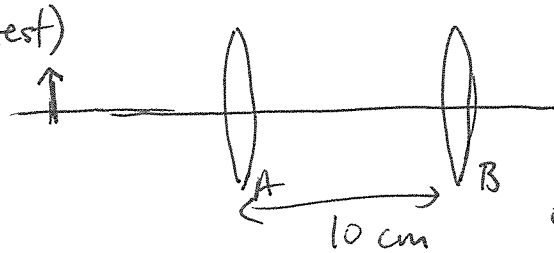
~~to determine f for a~~

Distances s and s' are positive if on the same side as the light rays, negative if opposite. ($s \rightarrow$ incoming, $s' \rightarrow$ outgoing)

$s' > 0 \rightarrow$ real image (can also happen for object in multi-lens setup)
$s' < 0 \rightarrow$ virtual image

magnification $M = -s'/s$ (sign determines if upright or inverted)

GRE #15 (test)



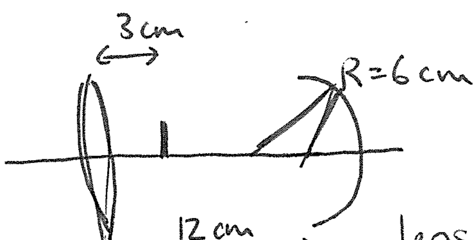
Two thin converging lenses A + B, each w/ focal length 6 cm, are 10 cm apart. If an object is placed 10 cm to the left of A, the final image is where?

lens 1: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \rightarrow \frac{1}{10} + \frac{1}{s'} = \frac{1}{6} \rightarrow s' = 15$ cm $\Rightarrow$ obj for lens #2

lens 2: $s = -5$ cm $\frac{1}{s'} = \frac{1}{6} + \frac{1}{5} = \frac{11}{30} \rightarrow s' = \frac{30}{11}$ cm to the right (real!)

A) 30 cm to the right $\boxed{\text{B) } 30/11 \text{ cm} \rightarrow}$ C) 30/10 cm $\rightarrow$ D) $\frac{30}{11}$ cm $\leftarrow$ E) $\frac{30}{10}$ cm $\leftarrow$

GRE #14



Converging lens of $f = 6$ cm is 12 cm to the left of concave mirror of $R = 6$ cm. An object is placed 3 cm to the right of the lens. How many real and virtual images are formed by the lens?

A) 1 real B) 1 virtual C) 2 real D) 2 virtual $\boxed{\text{E) 1 each}}$

Light can hit the object and go two directions, right or left.
For left, it goes through the lens only.

Case #1: $s = 3$ cm, $f = 6$ cm $s' = \left(\frac{1}{6} - \frac{1}{3}\right)^{-1} = -6$ cm virtual

(Also note object is inside focal length, meaning image is virtual.)

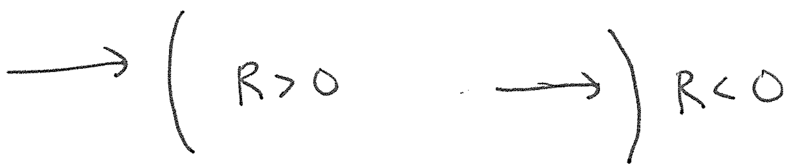
Case #2: light goes right, to mirror, and back through lens

mirror: $s' = \left(\frac{1}{3} - \frac{1}{9}\right)^{-1} = 4.5\text{ cm}$ (real image)

lens: $s' = \left(\frac{1}{6} - \frac{1}{7.5}\right)^{-1} = 30\text{ cm}$ (real image)

Lensmakers' equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$ for thin lens



for converging lens

we know $f > 0$ always, so $R_1 > 0$, $R_2 < 0$.

Thin films

(no time)