NSF GRFP Research Statement

<u>Motivation:</u> In the field of Quantum Mechanics, focus has shifted from merely understanding the quantum world to harnessing the bizarre behavior of quantum systems—specifically, for the rapidly growing fields of quantum computing and information processing, which promise exponential speedups over classical computations. I am proposing to harness quantum system behavior through a new approach to control: the optimal control of open quantum systems via the classically proven closed-loop control strategies of Stochastic Differential Dynamic Programming (SDDP), Forward Stochastic Differential Equations (FSDE), and Forward-Backward SDEs (FBSDE). These innovative methods are capable of accounting for the stochasticity often present in real systems, whether it be due to an unforeseen gust of wind or something more fundamental, as is the case for the quantum system. Control methods enabling the active manipulation of quantum systems are essential to bring the conjectured world of quantum technology to our reality.

<u>Obstacles:</u> ***Open Quantum Systems:*** The delicate nature of measuring a quantum system, coupled with the inherent uncertainty of a quantum state—two hallmarks of quantum theory—require classical control strategies to be substantially modified before becoming useful in the quantum regime. For a quantum control strategy to be widely useful for extended periods of time, it must also account for the *open* quantum system. The open quantum system is often ignored in quantum control literature [1]; however, it is imperative to consider the open quantum system when implementing a closed-loop control strategy. External interaction introduces noise in the open system's state, and the evolution of such a stochastic state is described by the Lindblad master equation, a partial differential equation. Stochastic optimal control strategies capable of rapidly propagating PDEs will thus be required to effectively control the open quantum system.

***Quantum Optimal Control*:** A quantum system must be controlled in order to be useful for application. Optimal control is preferred over other control schemes because it calculates a control sequence that minimizes some objective, such as the time or energy required to reach a desired state. Feedforward optimal control calculates the control sequence while minimizing some problem-dependent cost, given only the initial system state. In contrast, feed*back* optimal control uses information about the *current* state to actively adjust the applied control, thereby increasing the stability and robustness of the control in the presence of perturbations and noise. I propose the use of feedback optimal control to increase the versatility of the control strategy's application.

Quantum State Estimation: When dealing with a quantum system, access to the state of the system is limited. Projective measurement of a potentially mixed quantum state leads to its collapse, and information of the state prior to measurement is thereby destroyed. *Weak* measurement, in which partial information about the state is extracted over an interval of time, is less destructive to the state. However, this external interaction introduces noise, which necessitates the consideration of the stochastic open quantum system. A quantum observable can be continuously monitored through weak measurement [2]—which is propitious for the future of quantum feedback control. My research will consider weak measurement to enable closed-loop control strategies.

<u>Methods:</u> ***Stochastic Optimal Control***: Three potential classes of stochastic optimal feedback control have yet to be expanded to the quantum domain: SDDP, FSDE, and FBSDE. These approaches involve the minimization of the *value function*, governed by the Hamilton-Jacobi-Bellman (HJB) equation, which can be formulated for an open quantum system [3]. I have studied the application of Differential Dynamic Programming (DDP) to a deterministic closed quantum system (a Bose-Einstein condensate) over the past year, as described in my personal statement. This experience will prepare me to extend the theory to the stochastic open quantum system. Progress has been made in expanding DDP to control classical systems governed by nonlinear, stochastic dynamics (SDDP) [4]. Open quantum systems governed by the Linblad master equation make up one family of such systems, with the added complication of quantum state estimation. I will test SDDP's ability to control open quantum systems. Next, I will consider the sampling based methods of FSDE and FBSDE, which involve the propagation of stochastic trajectories without relying on linearization [5]. Such computations are parallelizable using graphics processing units (GPUs), so these methods may also be expanded to the quantum regime.

System Modelling: An obstacle to treating the open quantum system is learning to model stochasticity. Current research successfully uses neural networks to represent the density matrix of a potentially many-bodied open quantum system [6]. I will apply this approach to simulate the application of each closed-loop stochastic optimal control strategy to an open spin system.

Experiment: I will conduct physical tests of the algorithms at a lab that routinely handles quantum spin systems. I am applying to attend Caltech for my physics PhD, which has many atomic physics labs ideal for these experiments including The Institute for Quantum Information and Matter and the Quantum Optics Group. Additionally, I plan to forge a collaboration between my graduate and undergraduate institutions to employ the Quantum Testbed I maintain at Georgia Tech.

Timeline and Measures of Success:

Yr. 1) *Is it possible for DDP to converge when applied to a closed quantum system in simulation? Can SDDP, FSDE, and FBSDE be expanded to the quantum domain without losing stability and convergence assurances?* – Simulate the optimal control of a closed quantum system (a BEC) using DDP. Expand SDDP, FSDE, and FBSDE to handle open quantum systems.

Yr. 2) *With respect to performance and stability, what guarantees can be made about the newly Expanded algorithms when compared to open-loop optimal control methods? Can these methods converge when applied to an open quantum system in simulation?* – Perform simulated tests of the expanded algorithms to split a BEC.

Yr. 3) *Can these methods be used to manipulate a physical quantum system?* – Perform a physical test of one or more of the expanded stochastic control algorithms.

Success will be measured by comparing the convergence, stability, and robustness of these algorithms to those of quantum optimal control literature [7].

Intellectual Merit: The scientific community must achieve optimal control of quantum systems before reaping the full potential of quantum computers [8], which have the realistic potential of revolutionizing computing and current methods of security. Quantum optimal control would also increase precision in quantum metrology [9] and enable precise manipulation of chemical reactions [10]—healing our ozone layer would be a single application of this technology. Moreover, my research would expand optimal control capabilities that are of growing importance in today's age of autonomy, with applications in robotics, autonomous driving, and decision-making.

Broader Impacts: My research will provide an opportunity for me to mentor an undergraduate, as I was first introduced to quantum control, and to lead students (whether homeschooled, incarcerated, or others) to the edge of scientific knowledge. I also believe my research will enable the inevitable prevalence of quantum devices. As society realizes the practicality of quantum mechanics—often viewed as inapplicable to our daily experience—perhaps more people will be inspired to explore the stranger fields of STEM and encounter the beauty behind our universe.

[1] L. V. Damme, Q. Ansel, S. J. Glaser, and D. Sugny, *Phys. Rev. A* **95**, (2017). [2] K. Jacobs and D. A. Steck, *Contemp. Phys*. **47** , 279 (2006). [3] J. Gough, V. P. Belavkin, and O. G. Smolyanov. *J. Opt. B: Quantum Semiclass*. 7, S237 (2005) [4] E. Theodorou, Y. Tassa, and E. Todorov, *Proc. of the 2010 American Control Conf.* (2010). [5] M. Pereira, Z. Wang, I. Exarchos, and E. Theodorou. *Robotics: Science and Systems* (2019) [6] M. Schuld, I. Sinayskiy, and F. Petruccione, *Phys*. **12**, (2019). [7] U. Hohenester, P. K. Rekdal, A. Borzì, and J. Schmiedmayer, *Phys. Rev. A* **75**, (2007). [8] J. P. Dowling and G. J. Milburn, *Phil. Trans. R. Soc. Lond. A*, **361**, 1655 (2003). [9] J. K. Stockton, J. M. Geremia, A. C. Doherty, and H. Mabuchi, *Phys. Rev. A* **69**, (2004). [10] H. Rabitz, R. de Vivie-Riedle, M. Motzkus, and K. Kompa, *Science* **288**, 824 (2000).